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On the weak law for randomly indexed partial sums for arrays of random elements in martingale type p Banach spaces

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Abstract

For weighted randomly indexed sums of the form $\sum_{j=1}^{N_n} a_{nj}(V_{nj} - c_{nj})$ where $\{a_{nj}, j \geqslant 1, n \geqslant 1\}$ are constants, $\{V_{nj}, j \geqslant 1, n \geqslant 1\}$ are random elements in a real separable martingale type p Banach space, $\{N_n, n \geqslant 1\}$ are positive integer-valued random variables, and $\{c_{nj}, j \geqslant 1, n \geqslant 1\}$ are suitable conditional expectations, a general weak law of large numbers is established. No conditions are imposed on the joint distributions of the $\{V_{nj}, j \geqslant 1, n \geqslant 1\}$. Also, no conditions are imposed on the joint distributions of $\{N_n, n \geqslant 1\}$. Moreover, no conditions are imposed on the joint distributions of $\{N_n, n \geqslant 1\}$. Moreover, no conditions are imposed on the joint distribution of the sequence $\{V_{nj}, j \geqslant 1, n \geqslant 1\}$ and the sequence $\{N_n, n \geqslant 1\}$. The weak law is proved under a Cesàro type condition. The sharpness of the results is illustrated by an example. The current work extends that of Gut (1992), Hong and Oh (1995), Hong (1996), Kowalski and Rychlik (1997), Adler et al. (1997) and Sung (1998).

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1. Introduction

Consider an array of constants $\{a_{nj}, j \geqslant 1, n \geqslant 1\}$ and an array of random elements $\{V_{nj}, j \geqslant 1, n \geqslant 1\}$ defined on a probability space $(\Omega, \mathscr{F}, P)$ and taking values in a real separable Banach space $\mathscr{X}$ with norm $\|\cdot\|$. Let $\{c_{nj}, j \geqslant 1, n \geqslant 1\}$ be a "centering" array consisting of (suitably selected) conditional expectations and

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